

ASSESSMENT OF CABLE TENSION USING VIBRATION BASED METHODOLOGIES FOR WIRELESS STRUCTURAL HEALTH MONITORING

S. NANCY DEBORA¹, R. RAJA², SABITHA JANNET³, RISHIPAL REDDY⁴,
J. JACKSON VASANTH⁵ & U. ABHIJIT MENON⁶

¹Assistant professor, Department of Civil Engineering, Karunya University, Coimbatore, Tamil Nadu, India

^{2,3}Assistant professor, Department of Mechanical and Aerospace Engineering, Karunya University,
Coimbatore, Tamil Nadu, India

^{4,5,6}Scholars, Department of Civil Engineering, Karunya University, Coimbatore, Tamil Nadu, India

ABSTRACT

In Health Monitoring of the cable supported bridges, estimation of the cable tension influences the accuracy of the monitoring system. The purpose of this study is to prove the vibration based methodologies in conjunction with dynamic cable theory can be used to measure the tension force in cables. The idea being that the accelerations from the cable vibrations have embedded frequencies which can be extracted using Fast Fourier Transform and processed using the computer-aided software. After the accelerations are processed and the frequencies are extracted and organized, the cable tension can be calculated using dynamic cable theory and cable properties. Numerical studies were carried out on a cable with known tensile force using ABAQUS, to determine the natural frequencies. Adopting the flat taut string theory, the beam theory, Fang theory, Zui's theory and Wei-Xin Ren's cable force was assessed from natural frequency. A MATLAB script was developed to estimate the tension force of the cable for the above theories.

KEYWORDS: SHM, Cable tension, Frequency, FFT, Accelerations & MATLAB

Received: Feb 01, 2018; **Accepted:** Feb 23, 2018; **Published:** Mar 10, 2018; **Paper Id.:** IJMPERDAPR201867

1. INTRODUCTION

In these days, constructing long span bridges is a growing trend, and the record-breaking long-span bridges are frequently introduced. The emergence of high strength cables plays an important role in winning a great popularity of cable-supported bridges. Load cells or pressure meters were used to effectively monitor cables during construction. (Wang et al., 1999; Kim and Park, 2007). Elongation measurement is also one important method of measuring (Casas, 1994). Electromagnetic (EM) stress sensors were used to check the tension of cables post construction (Wang et al., 2005). The direct method of tension measurement is also used to measure. (Zui et al., 1996; Russell and Lardner, 1998). In this study, the performance of the existing vibration-based methods is investigated through numerical studies. The available vibration-based tension force estimation methods, a set of numerical studies on 2m cable with varying diameter and tensile force were carried out using ABAQUS, FEA software to determine the natural frequencies and mode shapes of the cables. A MATLAB script file is developed to automate the processing of the various vibration based methodologies to estimate the tension force through the extracted natural frequencies was validated.

2. VIBRATION - BASED METHODS

The relationship between frequencies and cable tension may be classified into five types based on the sag-extensibility and bending stiffness of a cable. In the flat taut string theory, neglecting both sag-extensibility and bending stiffness of the cable the relationship is given as:

$$T = 4mL^2 \left(\frac{f_n}{n} \right)^2 \quad (1)$$

Where T, m, L, and f_n denotes cable tension, mass density, length of cable, and the nth natural frequency, respectively. However, this simple formula is valid only for flat long slender single cable. Eq. (1) is helpful for the first approximation of tension force.

The second class makes use of a frequency formula for an axially loaded beam that considers the bending effect without sag-extensibility (Shimada et al., 1989):

$$T = 4mL^2 \left(\frac{f_n}{n} \right)^2 - \frac{EI\pi^2 n^2}{L^2} \quad (2)$$

Where the flexural rigidity of a cable is denoted by EI

The third category includes only the bending stiffness parameter (Fang et al., 2012). The sag effect on cable tension measurement was discussed by Zui et al. (1996) with introduction of parameters ξ and Γ

$$T = 4\pi^2 mL^2 \left(\frac{f_n}{\gamma_n^2} \right)^2 - \frac{EI}{L^2} \gamma_n^2 \quad (3)$$

Where $\gamma_n = n\pi + A\psi_n + B\psi_n^2$

$$A = -18.9 + 26.2n + 15.1n^2$$

$$B = \begin{cases} 290.0, & (n = 1) \\ 0, & (n \geq 2) \end{cases}$$

$$\psi_n = \frac{1}{\chi_n \gamma_n} = \sqrt{\frac{EI}{m\omega_n^2 l^4}}$$

$$\Gamma = \sqrt{\frac{wL}{128EA\delta^3 \cos^5 \theta} \left[\frac{0.31\xi + 0.5}{0.31\xi - 0.5} \right]},$$

$$\xi = l \sqrt{\frac{T}{EI}}$$

Where γ_n is the circular frequency of the cable, Zui et al. 1996 concluded that only the first-order mode can be used when $\Gamma \geq 3$, for cables with $\Gamma < 3$, the second or higher order mode should be used for the measurement of cable tension.

The fourth category includes both sag-extensibility (Γ) and bending stiffness effect (ξ). (Zui et al., 1996): The criteria for the use of the below-mentioned formulas are given as a). When $\Gamma > 3$, vibrations of the first-order mode are used because, the effects of cable sag and inclination are negligibly small even for first-order mode b). When $\Gamma \leq 3$, the effects of cable sag are large, namely, the values of Γ are small, it is desirable to use second-order mode for the measurement of cable force.

In the case that a cable has small sag ($3 \leq \Gamma$);

$$T = 4m(f_1 L)^2 \left[1 - 2.2 \frac{C}{f_1} - 0.55 \left(\frac{C}{f_1} \right)^2 \right] \quad (4a)$$

$$T = 4m(f_1 L)^2 \left[0.865 - 11.6 \left(\frac{C}{f_1} \right)^2 \right] (6 \leq \xi \leq 17) \quad (4b)$$

$$T = 4m(f_1 L)^2 \left[0.828 - 10.5 \left(\frac{C}{f_1} \right)^2 \right] (0 \leq \xi \leq 6) \quad (4c)$$

In case that a cable has large sag ($\Gamma \leq 3$)

$$T = m(f_2 L)^2 \left[1 - 4.40 \frac{C}{f_2} - 1.10 \left(\frac{C}{f_2} \right)^2 \right] (60 \leq \xi) \quad (5a)$$

$$T = m(f_2 L)^2 \left[1.03 - 6.33 \frac{C}{f_2} - 1.58 \left(\frac{C}{f_2} \right)^2 \right] (17 \leq \xi \leq 60) \quad (5b)$$

$$T = m(f_2 L)^2 \left[0.882 - 85.0 \left(\frac{C}{f_2} \right)^2 \right] (0 \leq \xi \leq 17) \quad (5c)$$

When higher modes are used,

$$T = \frac{4m}{n^2} (f_n L)^2 \left[1 - 2.20 \frac{C}{f_n} \right] (200 \leq \xi), (2 \leq n) \quad (6)$$

$$\text{Where } C = \sqrt{\frac{EI}{mL^4}}$$

δ = sag-to-span ratio. The sag-to-span ratio is often not available in practice because measuring the static shape of cable needs high cost. The last category includes the empirical formulae to estimate cable tension by considering the bending stiffness and sag extensibility parameter (Zui et al., 2004). The cable tension and the cable fundamental frequency relationship including the sag effect are given as:

$$T = 4m^2 f^2 l^2 ; (\lambda^2 \leq 0.17) \quad (7a)$$

$$T = \sqrt[3]{ml^2(4f^2 T^2 - 7.569mEA)} ; (0.17 \leq \lambda^2 \leq 4\pi^2) \quad (7b)$$

$$T = m^2 f^2 l^2 ; (4\pi^2 < \lambda^2) \quad (7c)$$

Empirical formulas to estimate cable tensions considering cable bending stiffness effect

$$\lambda^2 = \left(\frac{mgl}{H} \right)^2 \frac{EAl}{HL_e} T = 3.432m^2 f^2 l^2 - 45.191 \frac{EI}{l^2} ; (0 \leq \xi \leq 18) \quad (8a)$$

$$T = m \left(2lf - \frac{2.363}{l} \sqrt{\frac{EI}{m}} \right)^2 ; (18 < \xi \leq 210) \quad (8b)$$

$$T = 4m^2 f^2 l^2 ; (210 < \xi) \quad (8c)$$

$$\text{Where } L_e = l \left\{ 1 + \frac{1}{8} \left(\frac{mgl}{H} \right)^2 \right\}$$

The formulae are simple in form and they are very convenient for practical engineers to fast evaluate the cable tensions.

3. NUMERICAL STUDIES

A cable of nominal diameter of 7 mm and span of mm with varying loads and suspended cable with two hinged ends as shown in Figure.1 is considered. 0.3 kg/m is the mass density per unit length of the cable (m). The moment of inertia and the cross-sectional areas of the cable is $1.176 \times 10^{-10} \text{ m}^4$, $3.85 \times 10^{-5} \text{ m}^2$ respectively. The ratio of the span to dip of the cable is assumed as 10%. Table.1 shows the first three frequencies of the cable for the applied tensile force. Figure 1 shows modeled cable in ABAQUS. The nominal diameters are 2.5, 3, 4, 5,7mm. The ratio of the span to dip of the cable is assumed as 10%.

Table 1: Frequencies for the Applied Load

Applied Load (N)	Cable Tensile Force(N)	Natural Frequencies (Hz)		
		n=1	n=2	n=3
1000	1028	7.207	14.66	22.59
1500	1541	8.54	17.26	26.38
2000	2054	9.85	19.84	30.30
2500	2555	11	22.11	33.56

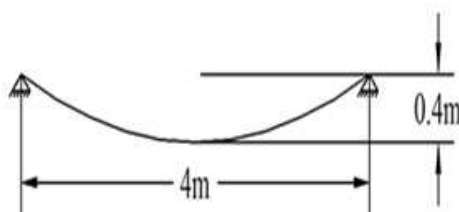


Figure 1: Suspended Cable Model

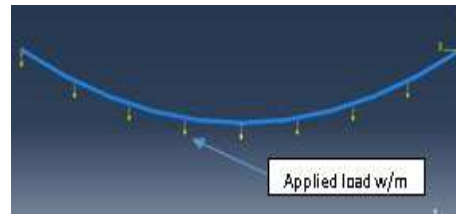


Figure 2: FEM Modeling with Applied Load

The FEM analysis of the suspended cable is carried out using ABAQUS software. The cable is modeled using 10 beam elements of type B32 (2- node, quadratic beam in space). The FEM model of the cable with applied load is shown in Figure.2. The modal analysis of the cable is carried out by varying the applied load from 1000N to 2500N in increments of 500N. To identify the parameters, a geometric nonlinear finite element analysis under the applied load is performed to calculate the static sag and distribution of the tension force. The vibration analysis is performed to calculate the natural frequencies and mode shapes of the cable. The same procedure is repeated by varying the applied load.

3.1. Studies on Cables with Varying Diameter and Loads

The modal analysis of the cable was carried out by varying the applied load from 500N to 2000N in increments of 500N. To identify the parameters, a geometric nonlinear finite element analysis under the applied load is performed to calculate the static sag and distribution of the tension force. The vibration analysis is performed to calculate the natural frequencies and the mode shapes of the cable through the extracted natural frequency

3.2. MATLAB Algorithm

A MATLAB script has been developed to automate the processing of various vibration based methodologies to estimate the tension force through the extracted natural frequency. Algorithm is shown in Figure 3

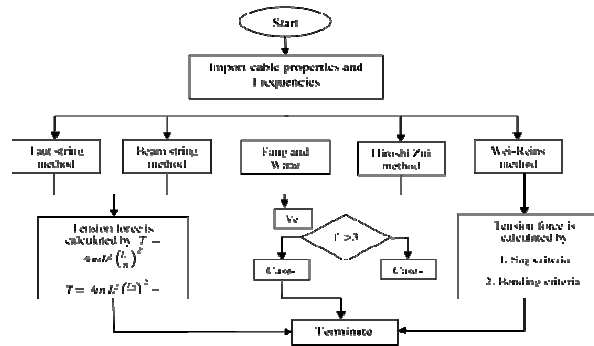


Figure 3: Flow Chart for Calculating Tensile Force

4. ESTIMATION OF TENSION FORCE

4.1. Studies on 7mm Cable with Varying Loads

For the extracted natural frequencies, the existing five vibration based methodologies are applied to estimate the tension forces through the developed MATLAB script.

Load case 1:1000N was carried out on the cable with an applied load of 1000N, where the sag to span ratio is 1.96×10^{-3} . Taut string and beam string theories utilize the symmetric mode of frequency to determine the tension force. In case of Fang and Zui's theories, the two condition parameters (\bar{I} and ξ) are taken into account to include the effect of sag and bending stiffness. Since $\bar{I} < 3$ and $\xi = 34.66$, the 1st anti-symmetric frequency is considered for tension force calculation. Hence the formulae 3 and 5a are adopted respectively. In load case 1, the cable ($\lambda_2=1.083$) has moderate sag and moderate ξ value, so by including the effect of sag, the tension force is calculated using the formula 7b (Wei-Xin).

Load case 2:1500N, on increasing the applied load, the sag to span ratio is incremented to 2.95×10^{-3} . The values of \bar{I} and ξ are 0.268, 41.12 respectively. Therefore formulae 3 and 5a (Fang and Zui's) are used in the calculation of tension force. As per Wei-Xin's theory the value of $\lambda_2 = 0.388$ and it represents the cable with moderate sag, the tension force is calculated using the formula 7b.

Load case 3:2000N the sag to span ratio of the cable is estimated as 3.92×10^{-3} . Sag extensibility and bending stiffness parameters of the cable are 0.196. and 47.2 respectively. The appropriate mode of frequency and formulae are selected to calculate the tension forces (Fang and Zui's). The value of $\lambda_2=0.166$, which represents cable with high sag. Hence the formula 7a is used in the calculation of tension force (Wei-Xin's).

Load case 4:2500N the sag to span ratio, sag extensibility parameters (\bar{I} , λ_2) and bending stiffness parameters (ξ) of the cable are 5×10^{-3} , 0.127, 0.084 and 53.04 respectively. This represents the cable with high sag and high bending stiffness effect, hence the appropriate mode of frequency and formulae are selected to calculate the tension forces.

The tension force evaluated by adopting various vibration based methodologies are listed in table: 2. The first row indicates the applied tensile force using line load feature in ABAQUS. The parenthesis in table: 2 shows the selected formulae and mode of frequency for the estimation of tensile force

Percentage of error with respect to evaluated tensile force is shown in figure: 4. Based on the estimated results, the taut string theory is limited to the cables with low sag-extensibility and bending stiffness. The % of error increases with increase in the sag to span ratio, however this theory can be used to calculate the initial value of T (7b). In case of the Zhi Fang and Beam,

Table 2: Evaluated Tensile Force

Load Case	Applied Load(N)	Cable Tensile Force (N)	Taut String Method	Beam String Method	Zhi Fang Method	Zui's Method	Wei-Xin Method
1	1000	1028	995(1,f1)	980(2,f1)	926.31(3,f2)	964.7(5b,f2)	983.6(7b,f1)
2	1500	1541	1400(1,f1)	1385(2,f1)	339(3,f2)	1380(5b,f2)	1410(7b,f1)
3	2000	2054	1862(1,f1)	1842(2,f1)	1874.2(3,f2)	1822(5b,f2)	1969.8(7a,f1)
4	2500	2555	2232.9(1,f1)	2314.6(2,f1)	2254(3,f2)	2329.9(5b,f2)	2459(7a,f1)

Note :() is the selected formulae and mode of frequency

String theory, which depends on the bending stiffness parameter of the cable, % of error is about 15%. For the practical formulae proposed by Zui et al [1], the ranges of error are also around 10%. Furthermore, the resulting tension force tends to be underestimated. The last category by Wei-Xin produces least error of about 4% except for load case 2, where the cable experiences a moderate sag ($\lambda_2 = 0.388$) but the initial value of T depends on the tension force estimated by taut string theory. The linear relationship between the cable tensile force and by various vibration based tension force estimation methodologies are plotted as shown in figure 4. Table 2 shows the evaluated tension force with the diameter of cable as 7mm and with varying load

5. STUDIES ON CABLES WITH VARYING DIAMETER AND LOADS

For comparative study, the percentage of error with respect to 1000N applied load for various diameters are considered. Figure 5 shows the results of the study. Based on the estimated results, the taut string theory and Wei Ren gives best results and the percentage of error is consistent with change in diameter. In case of beam string theory the percentage of error is proportional to the increase in diameter of the cable. In case of Fang and Zui's method the percentage error is observed to be maximum with respect to increase in the diameter of the cable. The bending and sag extensibility parameter for the applied load is the major cause for this increase in the percentage error.

6. RESULTS & DISCUSSIONS

6.1. Studies on 7mm Cable with Varying Loads

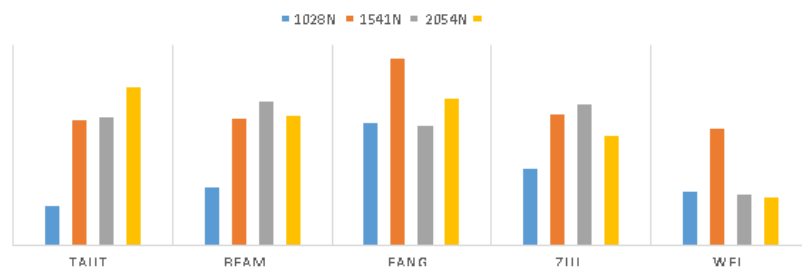
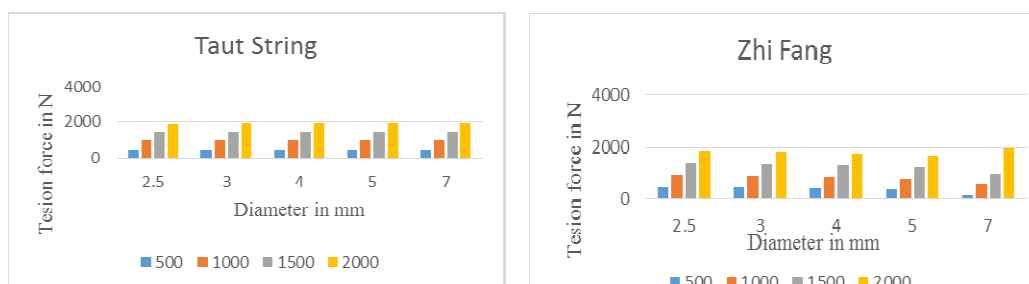


Figure 4: Percentage of Error with Respect to Evaluated Tensile Force

6.2. Studies on Cables with Varying Diameter and Loads



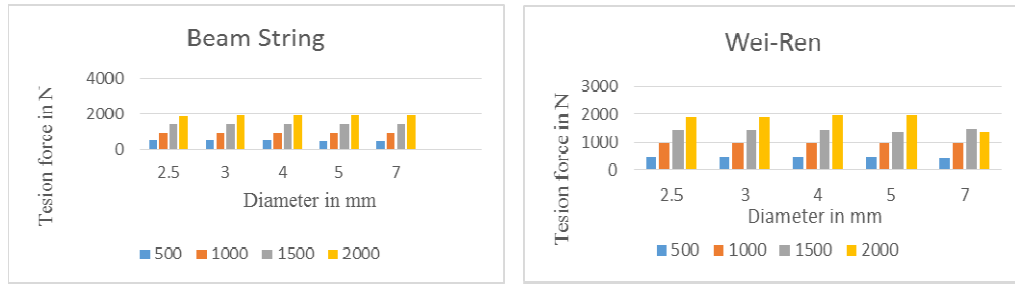


Figure 5: Shows the Evaluated Tension Force for Various Methodologies

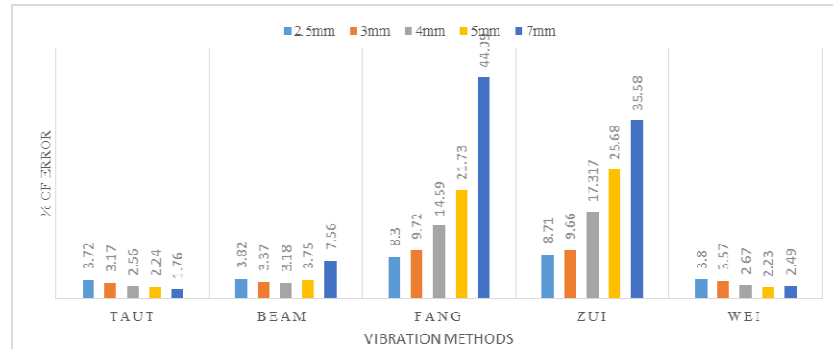


Figure 6: % Error Observed for 1000N Applied Load

7. CONCLUSIONS

Numerical studies were carried out on 7mm diameter cable of 4m, 0.4m span and dip respectively using ABAQUS software by varying the tensile force. Analysis was carried out to determine the vibration characteristics such as the frequencies of the cable. From the extracted frequencies, the tension force was evaluated. MATLAB script was developed to automate the estimation of tension force from the extracted frequencies. A comparative study of the existing methods was carried out and the corresponding percentage of errors were computed, the maximum percentage of error was around 15%. The processing of various vibration based methodologies to estimate the tension force. For comparative study, the percentage of error with respect to 1000N applied load for various diameters are considered. Figure 5 shows the results of the study. Based on the estimated results, the taut string theory and Wei Ren gives best results and the percentage of error is consistent with change in diameter. In case of beam string theory the percentage of error is proportional to the increase in diameter of the cable. In case of Fang and Zui's method, the percentage error is observed to be maximum with respect to increase in the diameter of the cable. The bending and sag extensibility parameter for the applied load is the major cause for this increase in the percentage error. The following conclusions were made 1) the string theory is a good tool for the first level of approximation due to its simplicity. 2) the empirical formulas by Wei-Xin has the most wide application range with reasonable accuracy. 3) the percentage of errors evaluated by various methodologies were around 10%.

REFERENCES

1. H. Zui, T. Shinke, Y.H. Namita, *Practical formulas for estimation of cable tension by vibration method*, *Journal of Structural Engineering*, ASCE 122 (6) (1996) 651–656.
2. Wei-Xin Ren, Gang Chen, Wei-Hua Hu, *Empirical formulas to estimate cable tension by cable fundamental frequency*, *Structural Engineering and Mechanics*, Vol. 20, No. 3 (2005) 363-380.

3. *Wei-Xin Ren, Hao-Liang Liu, Gang Chen, Determination of cable tensions based on frequency differences, Engineering Computations, Vol. 25 Iss 2 pp. 172 – 189.*
4. *Byeong Hwa Kim, Taehyo Park, Estimation of cable tension force using the frequency-based system identification method, Journal of Sound and Vibration, 304 (2007) 660–676.*
5. *Byeong Hwa Kim, Taehyo Park, Hyunyang Shin and Tae-Yang Yoon, A Comparative Study of the Tension Estimation Methods for Cable Supported Bridges, Steel Structures 7 (2007) 77-84.*
6. *Zhi Fang and Jian-qun Wang, Practical Formula for Cable Tension Estimation by Vibration Method, American Society of Civil Engineers. 10.1061/(ASCE) BE.1943-5592.0000200*